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ROBUST, ADAPTIVE, AND NETWORK CONTROL

Numerical Methods for Robust Performance Analysis of Linear Discrete-Time Polytopic Systems with Respect to Random Disturbances

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Abstract—Discrete linear polytopic systems affected by random correlated stationary disturbances are considered. New numerical methods for estimating of the anisotropic norm of a polytopic system using linear matrix inequalities are proposed.

Keywords: polytopic uncertainties, LMI, mean anisotropy, random processes, analysis

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1. INTRODUCTION

Mathematical models of control systems are designed on the basis of known physical laws, as well as measurable parameters of the plant. Technological tolerances and measurement errors in the control system may lead to a mismatch between the mathematical model and the real plant. In some cases, this mismatch is significant and may lead to loss of system performance, and to the loss of stability of the closed-loop system. Thus, analysis and control problems subject to inexact knowledge about the parameters of mathematical models, called robust analysis and control problems, arise.

Depending on the initial assumptions regarding the type of uncertainties of the control system, there are various approaches to the analysis of its robust properties. One of the popular means of describing uncertainties in linear systems is polytopic uncertainty. This uncertainty is characterized by the fact that the unknown parameters of the system lie on the given simplex. If a system with polytopic uncertainty is lienar time invariant one, then in the literature it is called a polytopic system. For discrete-time linear systems, there are many methods for checking robust stability [1–4]. Papers [1–3] are devoted to the study of the stability of systems with polytopic time-invariant and time varying uncertainties using parametric Lyapunov functions. In [4] the results of robust analysis of polytopic systems using of linear matrix inequalities are presented. The results are given in terms of nonparametric matrix inequalities.

Along with the problems of studying robust stability of uncertain systems, one of the important aspects analysis of control systems is the ability to suppress external disturbances. Thus, in the literature it is known methods for analyzing the quality of suppression of external disturbances in terms of $\mathcal{H}_2$- and $\mathcal{H}_\infty$-norms [5]. Assuming that correlated random disturbances act at the input of the system, an anisotropy-based approach can be used to analyze the quality of its suppression by the system [6–8]. Feature of anisotropy-based approach is to study the quality of system performance under impact of correlated stationary random disturbances with a known mean anisotropy level. Methods of anisotropy-based analysis and control of polytopic systems were studied in [9–11].

In [9] parametric version of the anisotropy-based bounded real lemma, one of the results of non-parametric numerical analysis anisotropy-based performance analysis was obtained in [10], [11] is devoted to solving problem of anisotropy-based state-feedback control design with closed-loop pole placement.

This paper proposes numerical methods for solving the problem of anisotropy-based analysis for polytopic systems using linear matrix inequalities. All these methods are derived from parametric anisotropy-based bounded real lemma. The degree of conservatism of the obtained conditions is analyzed, and also estimates of their computational complexity are given.

2. PROBLEM STATEMENT

Consider linear system with state space representation as

$$x(k+1) = A(\Theta)x(k) + B_w(\Theta)w(k), \tag{1}$$

$$y(k) = C(\Theta)x(k) + D_w(\Theta)w(k), \tag{2}$$

where $x(k) \in \mathbb{R}^n$ is a state, $w(k) \in \mathbb{R}^m$ is external random disturbance with zero mean and bounded mean anisotropy level $\overline{\mathbf{A}}(W) \leqslant a$ $(a \geqslant 0)$, $y(k) \in \mathbb{R}^p$ is output.

Matrices $A(\Theta)$, $B_w(\Theta)$, $C(\Theta)$, $D_w(\Theta)$ are defined from the expressions

$$\begin{aligned} A(\Theta) &= \sum_{i=1}^{r} \theta_i A_i, \quad B_w(\Theta) = \sum_{i=1}^{r} \theta_i B_{wi}, \\ C(\Theta) &= \sum_{i=1}^{r} \theta_i C_i, \quad D_w(\Theta) = \sum_{i=1}^{r} \theta_i D_{wi}, \end{aligned} \tag{3}$$

with known constant matrices A_i, B_{wi}, C_i, D_{wi} of appropriate dimensions and vector Θ of unknown parameters which satisfies relations

$$\sum_{i=1}^{r} \theta_i = 1, \quad \theta_i \geqslant 0, \quad \theta_i \in \mathbb{R}, \quad \forall i = \overline{1, r}. \tag{4}$$

Mean anisotropy characterizes a measure of difference between Gaussian random sequence and white Gaussian noise with zero mean and identity covariance (we call it standard) in terms of relative entropy and is calculated using the formula

$$\overline{\mathbf{A}}(W) = -\frac{1}{4\pi} \int_{-\pi}^{\pi} \ln\det \frac{mS_w(\omega)}{\frac{1}{2\pi}\int_{-\pi}^{\pi} \operatorname{Trace} S_w(\lambda)d\lambda} d\omega, \tag{5}$$

where $S_w(\omega)$ is a spectral density of sequence $W = \{w(k)\}_{k\in\mathbb{Z}}$.

Thus, parameter $a \geqslant 0$ defines the set of all Gaussian signals whose measure of difference from standard Gaussian noise defined by expression (5), does no exceed value of a. It should be noted that mean anisotropy functional is nonnegative and goes to zero if W is standard Gaussian noise [8].

Denote the set of all parameters Θ, satisfying (3) and (4), by $\mathfrak{Q}$ and consider the mapping $Y = F_\Theta W$, defined by expressions (1)–(2).

Definition 1. Anisotropic norm of polytopic system (1)–(4) is norm of operator F_Θ, defined by expression

$$|\!|\!| F_\Theta |\!|\!|_a = \sup_{\Theta\,\in\,\mathfrak{Q}} \ \sup_{W:\ A(W)\leqslant a} \frac{\|Y\|_{\mathcal{P}}}{\|W\|_{\mathcal{P}}}, \tag{6}$$

where

$$\|W\|_{\mathcal{P}} = \sqrt{\lim_{N\to\infty} \frac{1}{2N+1} \sum_{k=-N}^{N} \mathbf{E}|w(k)|^2}$$

is power norm of signal W.

One of the most important features of anisotropic norm is that it lies between scaled $\mathcal{H}_2$-norm and $\mathcal{H}_\infty$-norm, i.e. [8]

$$\frac{\|F_\Theta\|_2^2}{m} \leqslant |\!|\!|F_\Theta|\!|\!|_a^2 \leqslant \|F_\Theta\|_\infty^2.$$

It mean that by varying value of mean anisotropy a from 0 to ∞, one can reduce or expand the set of random signals, selecting the most favorable bandwidth and stability margins of the system in the range between $\mathcal{H}_2$- and $\mathcal{H}_\infty$-norms.

In problem of robust anisotropy-based analysis of polytopic systems it's necessary to obtain conditions for checking robust stability and anisotropic norm bounds of open-loop system (1)–(2) for known mean anisotropy level $a \geqslant 0$ and given scalar $\gamma > 0$. Thus, the problem is formulated as follows.

Problem 1. For known mean anisotropy level $a \geqslant 0$ of random external disturbance $w(k)$ and given scalar $\gamma > 0$ the problem is to check:

1) if the system robustly stable;

2) if the condition holds

$$|\!|\!|F_\Theta|\!|\!|_a < \gamma.$$

Known results which are necessary for the further exposition are listed below. Let us consider the system with known parameters, for which all of the vectors and matrices dimensions coincide with ones in system (1)–(2):

$$x(k+1) = Ax(k) + B_w w(k), \tag{7}$$

$$y(k) = Cx(k) + D_w w(k). \tag{8}$$

Now provide formulation of anisotropy-based bounded real lemma in terms of LMI [13].

Lemma 1. *System* (7)–(8) *is stable and its anisotropic norm for given mean anisotropy level of external disturbance* $a \geqslant 0$ *is bounded by scalar* $\gamma > 0$, *if there exist such matrices* $X > 0$, $Y > 0$, $\Phi > 0$, *and scalar* $\mu > \gamma^2$, *for which the following relations hold true:*

$$\mu - \left(\mathrm{e}^{-2a} \det \Phi\right)^{1/q} < \gamma^2, \tag{9}$$

$$\begin{bmatrix} \Phi - \mu I_m & \star & \star \\ B_w & -Y & \star \\ D_w & 0 & -I_p \end{bmatrix} < 0, \tag{10}$$

$$\begin{bmatrix} -X & \star & \star & \star \\ 0 & -\mu I_m & \star & \star \\ A & B_w & -Y & \star \\ C & D_w & 0 & -I_p \end{bmatrix} < 0, \tag{11}$$

$$XY = I_n. \tag{12}$$

3. PROBLEM SOLUTION

3.1. Parametric Anisotropy-Based Bounded Real Lemma

Let us formulate parametric conditions for anisotropy-based analysis of a polytopic system (1)–(2), on the basis of which the main results of this paper will be obtained.

Theorem 1. *System* (1)–(2) *is robustly stable and its anisotropic norm does not exceed given scalar value* $\gamma > 0$ *for known mean anisotropy level* $a \geqslant 0$ *if there exist such matrices* $P(\Theta) > 0$, $\Psi(\Theta) > 0$, *nonsingular matrices* $G_1(\Theta)$, $G_2(\Theta)$ *and scalar* $\eta > \gamma^2$, *such that the following inequalities hold true*

$$\eta - \left(\mathrm{e}^{-2a} \det \Psi(\Theta)\right)^{1/m} < \gamma^2, \tag{13}$$

$$\begin{bmatrix} \Psi(\Theta) - \eta I_m & \star & \star \\ G_1(\Theta) B_w(\Theta) & L_1(\Theta) & \star \\ D_w(\Theta) & 0 & -I_p \end{bmatrix} < 0, \tag{14}$$

$$\begin{bmatrix} -P(\Theta) & \star & \star & \star \\ 0 & -\eta I_m & \star & \star \\ G_2(\Theta)A(\Theta) & G_2(\Theta)B_w(\Theta) & L_2(\Theta) & \star \\ C(\Theta) & D_w(\Theta) & 0 & -I_p \end{bmatrix} < 0, \tag{15}$$

where $L_1(\Theta) = -G_1(\Theta) - G_1^{\mathrm{T}}(\Theta) + P(\Theta)$ *and* $L_2(\Theta) = -G_2(\Theta) - G_2^{\mathrm{T}}(\Theta) + P(\Theta)$, *for each* $\Theta \in \mathfrak{Q}$.

The proof of the theorem is listed in Appendix.

Conditions of Theorem 1 depend on parameters Θ explicitly. Existence of any parametric matrices $\Psi(\Theta)$, $P(\Theta)$, $G_1(\Theta)$, and $G_2(\Theta)$, which satisfy all the conditions of the Theorem 1, would allow to check robust stability of the system and establish the fact that its anisotropic norm is bounded by γ for known mean anisotropy level $a \geqslant 0$ of input disturbance W. There is currently no formal method for determining the exact type matrices $P(\Theta)$ as a function of the parameter vector Θ. In scientific literature such function $P(\Theta)$ is called parametric Lyapunov matrix [1, 2, 4]. Similar statement holds for the rest parametric matrices. Unfortunately, such parametric dependence may substantially complicate analysis of initial plant. It is possible to reduce numerical complexity of the algorithm by introducing supplementary restrictions, for example, by using different approximations of matrices $\Psi(\Theta)$, $P(\Theta)$, $G_1(\Theta)$ and $G_2(\Theta)$. On the one hand, this approach allows to get rid of explicit appearance of parameter vector Θ, on the other hand, it bring some conservatism. Below we present several methods of nonparametric anisotropy-based analysis of the polytopic system (1)–(2) depending on various approximations.

3.2. Nonparametric Variations of Anisotropy-Based Bounded Real Lemma

Let $\Psi(\Theta) = \Psi$, $G_1(\Theta) = G_1$, $G_2(\Theta) = G_2$, $P(\Theta) = P$. Then parameters θ_i can be factorized in expressions (14)–(15). The following result is obtained directly.

Theorem 2. *System* (1)–(2) *is robustly stable and its anisotropic norm does not exceed given scalar value* $\gamma > 0$ *for known mean anisotropy level* $a \geqslant 0$ *if there exist such matrices* $P > 0$, $\Psi > 0$, *nonsingular matrices* G_1*,* G_2*, and scalar* $\eta > \gamma^2$*, for which the following inequalities hold true:*

$$\eta - \left(\mathrm{e}^{-2a} \det \Psi\right)^{1/m} < \gamma^2, \tag{16}$$

$$\begin{bmatrix} \Psi - \eta I_m & \star & \star \\ G_1 B_{wi} & L_1 & \star \\ D_{wi} & 0 & -I_p \end{bmatrix} < 0, \tag{17}$$

$$\begin{bmatrix} -P & \star & \star & \star \\ 0 & -\eta I_m & \star & \star \\ G_2 A_i & G_2 B_{wi} & L_2 & \star \\ C_i & D_{wi} & 0 & -I_p \end{bmatrix} < 0, \tag{18}$$

where $L_1 = -G_1 - G_1^{\mathrm{T}} + P$*,* $L_2 = -G_2 - G_2^{\mathrm{T}} + P$ *and* $i = \overline{1, r}$*.*

The proof is trivial and is not given in the paper. Theorem 2 represents the simplest and most conservative solution to the Problem 1.

Now we will use linear approximation for parametric Lyapunov matrix and some auxiliary variables.

Theorem 3. *System* (1)–(2) *is robustly stable and its anisotropic norm does not exceed given scalar value* $\gamma > 0$ *for known mean anisotropy level* $a \geqslant 0$ *and all possible uncertainties which satisfy* (3)–(4)*, if there exist matrices* $P_i > 0$*,* $\Psi > 0$*, nonsingular matrices* G_{1i}*,* G_{2i}*, and scalar value* $\eta > \gamma^2$*, for which the following inequalities hold true:*

$$\eta - \left(\mathrm{e}^{-2a} \det \Psi\right)^{1/m} < \gamma^2, \tag{19}$$

$$\begin{bmatrix} \Psi - \eta I_m & \star & \star \\ G_{1i} B_{wi} & -G_{1i} - G_{1i}^{\mathrm{T}} + P_i & \star \\ D_{wi} & 0 & -I_p \end{bmatrix} < 0, \tag{20}$$

$$\begin{bmatrix} -P_i & \star & \star & \star \\ 0 & -\eta I_m & \star & \star \\ G_{2i} A_i & G_{2i} B_{wi} & -G_{2i} - G_{2i}^{\mathrm{T}} + P_i & \star \\ C_i & D_{wi} & 0 & -I_p \end{bmatrix} < 0, \tag{21}$$

$$\begin{bmatrix} \Psi - \eta I_m & \star & \star \\ G_{1i} B_{wj} & -G_{1i} - G_{1i}^{\mathrm{T}} + P_i & \star \\ D_{wj} & 0 & -I_p \end{bmatrix} + \begin{bmatrix} \Psi - \eta I_m & \star & \star \\ G_{1j} B_{wi} & -G_{1j} - G_{1j}^{\mathrm{T}} + P_j & \star \\ D_{wi} & 0 & -I_p \end{bmatrix} < 0, \tag{22}$$

$$\begin{bmatrix} -P_i & \star & \star & \star \\ 0 & -\eta I_m & \star & \star \\ G_{2i} A_j & G_{2i} B_{wj} & -G_{2i} - G_{2i}^{\mathrm{T}} + P_i & \star \\ C_j & D_{wj} & 0 & -I_p \end{bmatrix} + \begin{bmatrix} -P_j & \star & \star & \star \\ 0 & -\eta I_m & \star & \star \\ G_{2j} A_i & G_{2j} B_{wi} & -G_{2j} - G_{2j}^{\mathrm{T}} + P_j & \star \\ C_i & D_{wi} & 0 & -I_p \end{bmatrix} < 0, \tag{23}$$

where $i, j = \overline{1, r}$*,* $i < j$*.*

The proof of the theorem is listed in Appendix.

Conditions derived in Theorem 3 do not depend on the parameter vector Θ and allow us to estimate anisotropic norm of the polytopic system by checking fulfilment of $2r + r(r-1) + 1$ inequalities. The number of inequalities as well as decision variables can be reduced by increasing the

Table 1. Estimation of numerical complexity of analysis methods

Method	Number of inequalities	Number of decision variables	Number of unknown parameters
Theorem 2	$2r+2$	6	$1+\frac{m^2+m}{2}+\frac{5n^2+n}{2}$
Theorem 3	$2r+r(r-1)+2$	$2+3r$	$1+\frac{m^2+m}{2}+r\frac{5n^2+n}{2}$
Theorem 4	$2r+\frac{r(r-1)}{2}+2$	$2+2r$	$1+\frac{m^2+m}{2}+r\frac{3n^2+n}{2}$

conservatism of the estimation, taking into account the fact that $\Phi(\Theta) = P^{-1}(\Theta)$. Let us formulate a theorem.

Theorem 4. *System* (1)–(2) *is robustly stable and its anisotropic norm is strictly less than scalar* $\gamma > 0$ *for known mean anisotropy level* $a \geqslant 0$ *and all possible inequalities, satisfying* (3)–(4), *if there exist such matrices* $\Phi_i > 0$, $\Psi > 0$, *nonsingular matrices* G_i, *and scalar* $\eta > \gamma^2$, *for which the following inequalities hold true:*

$$\eta - \left(\mathrm{e}^{-2a}\det\Psi\right)^{1/m} < \gamma^2, \tag{24}$$

$$\begin{bmatrix} \Psi-\eta I_m & \star & \star \\ B_{wi} & -\Phi_i & \star \\ D_{wi} & 0 & -I_p \end{bmatrix} < 0, \tag{25}$$

$$\begin{bmatrix} -G_i-G_i^{\mathrm{T}}+\Phi_i & \star & \star & \star \\ 0 & -\eta I_m & \star & \star \\ A_iG_i & B_{wi} & -\Phi_i & \star \\ C_iG_i & D_{wi} & 0 & -I_p \end{bmatrix} < 0, \tag{26}$$

$$\begin{bmatrix} -G_i-G_i^{\mathrm{T}}+\Phi_i & \star & \star & \star \\ 0 & -\eta I_m & \star & \star \\ A_jG_i & B_{wj} & -\Phi_i & \star \\ C_jG_i & D_{wj} & 0 & -I_p \end{bmatrix} + \begin{bmatrix} -G_j-G_j^{\mathrm{T}}+\Phi_j & \star & \star & \star \\ 0 & -\eta I_m & \star & \star \\ A_iG_j & B_{wi} & -\Phi_j & \star \\ C_iG_j & D_{wi} & 0 & -I_p \end{bmatrix} < 0, \tag{27}$$

where $i, j = \overline{1, r}$, $i < j$.

The proof of the theorem is listed in Appendix.

Conditions derived in Theorem 4 allow to estimate anisotropic norm of polytopic system by checking of fulfilment of $2r + \frac{r(r-1)}{2} + 2$ inequalities. Data on the computational complexity of using each of the theorems formulated above are given in Table 1.

To estimate the anisotropic norm of the system (1)–(2) one can solve the problem of minimizing the variable γ on the set of convex constraints specified by the theorems derived above.

Unfortunately, analytical methods do not allow to evaluate the degree of conservatism of conditions obtained in Theorems 3 and 4. The degree of conservatism of the conditions can only be assessed for specific examples using numerical tools. These tools can be developed based on the Theorem 1. Consider the grid method analysis of polytopic systems based on Theorem 1. The algorithm can be presented as follows.

Algorithm 1 (grid method)

Step 1. Set mean anisotropy level $a \geqslant 0$ and step of grid h. Define set Ω, lying inside unit cube of dimensions $\mathbb{R}^{r-1}$ and consisting of mesh points. Fix parameter Θ, by setting first $(r-1)$

components coordinates of a point from the set Ω, the last component is calculated by formula

$$\theta_r = 1 - \sum_{i=1}^{r-1} \theta_i = 1.$$

Step 2. Set $k = 1$.

Step 3. While $k \leqslant N$, choose element from the set Ω_k, fix system matrices $A_k = \sum_{i=1}^{r} \theta_i A_i$, $B_k = \sum_{i=1}^{r} \theta_i B_{wi}$, $C_k = \sum_{i=1}^{r} \theta_i C_{zi}$, $D_k = \sum_{i=1}^{r} \theta_i D_{zwi}$.

Step 4. For fixed values A_k, B_k, C_k, D_k solve optimization problem:

$$\gamma_k^2 = \min \gamma^2$$

on the set of variables $\{\eta, \gamma^2, P, \Psi, G_1, G_2\}$, satisfying (9)–(11).

Step 5. If system of matrix inequalities is not feasible at the Step 4, then initial plant is not stable for given parameter values, algorithm stops. If the solution is found, then value $\gamma_* = \max\{\gamma_k, \gamma_{k-1}\}$ is calculated. If $k < N$, then $k = k + 1$, and go to Step 4. If $k = N$, then go to Step 6.

Step 6. Upper bound of anisotropic norm is defined as γ_*.

One of the disadvantages of this method is that a sufficiently large grid step will not allow one to estimate the anisotropic norm with satisfactory accuracy and give an answer about the stability of the system. Therefore, it is recommended to first check the system for robust stability using one of the existing methods.

4. NUMERICAL EXAMPLE

In the following example, we will investigate the degree of conservatism of the methods for estimating the anisotropic norm of a polytopic system, formulated in the Theorems 2–4.

Example 1. Let the system be given by the following matrices:

$$A_1 = \begin{bmatrix} 0.9 & -0.7 \\ 0.5 & -0.3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 1 \\ -0.5 & -0.7 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0.7 & 0.4 \\ -0.5 & -0.5 \end{bmatrix},$$

$$B_{w1} = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}, \quad B_{w2} = \begin{bmatrix} -0.5 \\ 2 \end{bmatrix}, \quad B_{w3} = \begin{bmatrix} 0 \\ -2 \end{bmatrix},$$

$$C_1 = C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 1 & 0.3 \end{bmatrix}, \quad D_{w1} = 0, \quad D_{w2} = 0.1, \quad D_{w3} = -0.1.$$

Note that this system is stable for all possible values of the parameters Θ. To assess the degree of conservatism of methods, proposed in Theorems 2–4, we will use the grid method for analyzing the system with grid step $h = 0.01$. Figures 1–3 illustrate the results of minimizing the value of γ at various grid nodes. When calculating the norm, Theorem 1 was used for selected numerical values of the parameter vector Θ at various grid nodes.

As can be seen in figures, the double supremum (6) for different values of mean anisotropy a is reached at points Θ which do not coincide with each other. The variation of the norm occurs smoothly and without jumps. Checking stability conditions and an attempt to estimate the anisotropic norm using the Theorem 2 leads to an infeasible problem, therefore, numerical results are given only for Theorems 3 and 4. Results of numerical experiments for calculating anisotropic norm of the system are given in Table. 2.

The conditions of the Theorem 2 are the most conservative, which led to an infeasible problem. Theorems 3 and 4 allow us to numerically estimate the anisotropic norm of a given system using linear matrix inequalities. It can be seen from Table 2, the conditions of the Theorem 4 provide

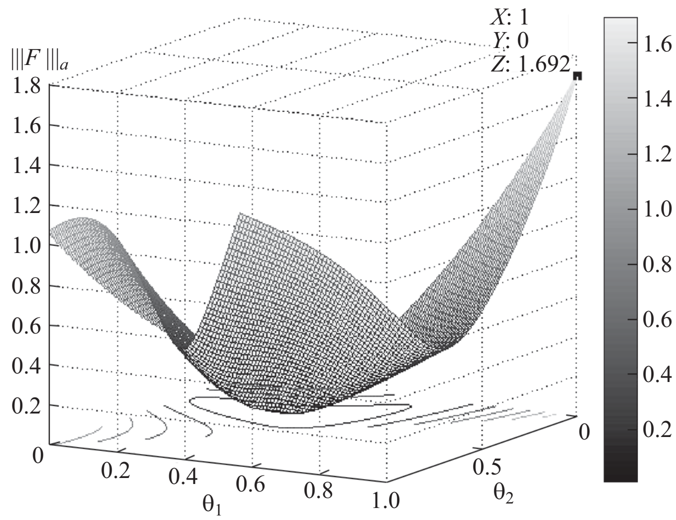

Fig. 1. Dependence of the minimum value γ on the parameters Θ at $a = 0$.

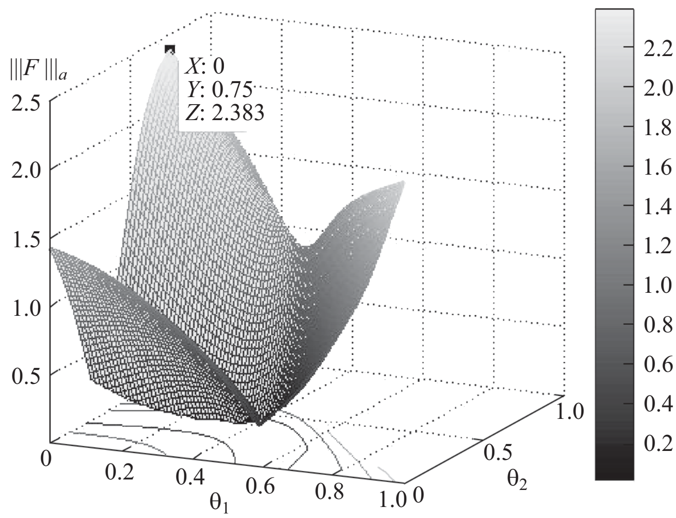

Fig. 2. Dependence of the minimum value γ on the parameters Θ at $a = 0.5$.

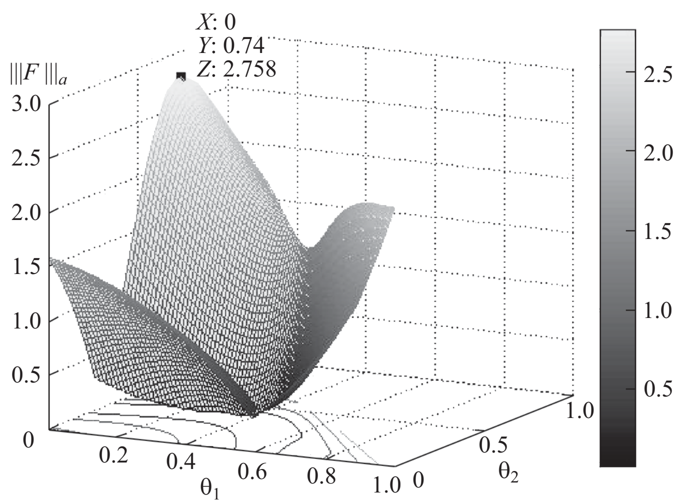

Fig. 3. Dependence of the minimum value γ on the parameters Θ at $a = 1.5$.

Table 2. Results of calculating the anisotropic norm in Example 1

Mean anisotropy a	0	0.1	0.5	1	1.5	100
$\|\|\|F_\Theta\|\|\|_a$ based on Theorem 1	1.6921	1.9258	2.3825	2.6616	2.7564	2.8100
$\|\|\|F_\Theta\|\|\|_a$ based on Theorem 3	4.6495	6.2913	7.9585	8.6163	8.8423	8.9707
$\|\|\|F_\Theta\|\|\|_a$ based on Theorem 4	6.7304	8.1552	9.0582	9.3701	9.4742	9.5327

Table 3. Results of calculating the anisotropic norm in Example 2

Mean anisotropy a	0	0.1	0.3	0.7	1.5	10
$\|\|\|F_\Theta\|\|\|_a$ based on Theorem 2	0.0771	1.1699	1.9277	2.6854	3.3354	3.7838
$\|\|\|F_\Theta\|\|\|_a$ based on Theorem 3	0.0728	0.3581	0.5827	0.8088	1.0032	1.1375
$\|\|\|F_\Theta\|\|\|_a$ based on Theorem 4	0.0727	0.3579	0.5820	0.8083	1.0028	1.1366

more conservative results. Despite this, the asymptotic behavior of the anisotropic norm for the given numerically implementable methods is preserved with a significantly lower computational complexity. Thus, these methods can be used to estimate the anisotropy-based performance of polytopic systems.

Example 2. Consider now mathematical model of damped oscillations of a spring pendulum:

$$\dot{x}(t) = Ax(t) + B_w w(t),$$
$$y(t) = x_1(t) + D_w w(t).$$

Here

$$A = \begin{bmatrix} 0 & 1 \\ -\omega^2 & -2\xi\omega \end{bmatrix},$$

where ω is natural frequency of the system, ξ is attenuation coefficient, $x_1(t)$ is pendulum's center of mass position, and $x_2(t)$ is pendulum's center of mass speed.

Disturbance $w(t) \in \mathbb{R}^2$ consists of external disturbance, acting on position $x_1(t)$, and measurement noise. Then

$$B_w = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad D_w = \begin{bmatrix} 0 & 0,1 \end{bmatrix}.$$

Let the system parameters are $\xi = 0.1$, $\omega \in [4.5;\ \ 5.2]$.

Initial plant given continuous time is discretized using zero order hold as

$$A^d = \mathrm{e}^{A^f h}, \quad B^d_w = \int\limits_0^h \mathrm{e}^{A^f(h-\tau)} B^w d\tau, \tag{28}$$

where h is discretization step.

Initial continuous plant was discretized with discretization step $h = 10^{-3}$ sec. The following parameters were obtained:

$$A^d_1 = \begin{bmatrix} 1 & 0.0010 \\ -0.0202 & 0.9991 \end{bmatrix}, \quad A^d_2 = \begin{bmatrix} 1 & 0.0010 \\ -0.0270 & 0.9989 \end{bmatrix},$$
$$B^d_{w1} = 10^{-3} \times \begin{bmatrix} 1 & 0 \\ -0.0101 & 0 \end{bmatrix}, \quad B^d_{w2} = 10^{-3} \times \begin{bmatrix} 1 & 0 \\ -0.0135 & 0 \end{bmatrix},$$
$$C_1 = C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D_{w1} = D_{w2} = \begin{bmatrix} 0 & 0.1 \end{bmatrix}.$$

Note that initial system is stable. The results of calculating of the anisotropic norm for a spring pendulum are summarized in Table 3.

5. CONCLUSIONS

In this paper, conditions for the boundedness of the anisotropic norm of a linear polytopic system are obtained in terms of linear matrix inequalities. Various options for nonparametric estimation were considered anisotropic norm, and also analyzed the estimation accuracy and computational complexity of these methods. The conditions are convex and formulated in terms of matrix inequalities, the number of which depends on the number of vertices of the polytope.

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APPENDIX

Proof of Theorem 1. The proof of the theorem consists of two parts. In the first one we will obtain conditions under which the polytopic the system (1)–(2) is robustly stable, and its $\mathcal{H}_\infty$-norm is bounded some number $\sqrt{\eta}$, i.e. $F_\Theta \in {\mathcal{H}_\infty}^{p\times m}$. In the second part of the proof we obtain conditions for the boundedness of the anisotropic norm for the robustly stable system $F_\Theta \in {\mathcal{H}_\infty}^{p\times m}$.

Consider the following parametric function as Lyapunov function candidate

$$V(k) = x^{\mathrm{T}}(k)P(\Theta)x(k), \quad P(\Theta) > 0. \tag{A.1}$$

Since we first require to prove the stability of the system and the boundedness of its $\mathcal{H}_\infty$-norm, then, to simplify calculations and without loss of generality, we assume that $W = \{w(k)\}_{k\,\in\,\mathbb{Z}} \in \mathcal{L}_2$. The difference between $V(k+1)$ and $V(k)$ is determined by the formula

$$V(k+1) - V(k) = x^{\mathrm{T}}(k+1)P(\Theta)x(k+1) - x^{\mathrm{T}}(k)P(\Theta)x(k). \tag{A.2}$$

Now we consider the expression:

$$\begin{aligned} &V(k+1) - V(k) + z^{\mathrm{T}}(k)z(k) - \eta w^{\mathrm{T}}(k)w(k) \\ &\quad = \{\text{substitute}\, x(k+1) = A(\Theta)x(k) + B_w(\Theta)w(k) \text{ and } z(k) = C(\Theta)x(k) + D_w(\Theta)w(k)\} \\ &\quad = \begin{bmatrix} x^{\mathrm{T}}(k) & w^{\mathrm{T}}(k) \end{bmatrix} \Bigg(\begin{bmatrix} A(\Theta) & B_w(\Theta) \end{bmatrix}^{\mathrm{T}} P(\Theta) \begin{bmatrix} A(\Theta) & B_w(\Theta) \end{bmatrix} \\ &\qquad + \begin{bmatrix} C(\Theta) & D_w(\Theta) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} C(\Theta) & D_w(\Theta) \end{bmatrix} - \begin{bmatrix} P(\Theta) & 0 \\ 0 & \eta I_m \end{bmatrix} \Bigg) \begin{bmatrix} x(k) \\ w(k) \end{bmatrix}. \end{aligned} \tag{A.3}$$

Thus, inequality

$$V(k+1) - V(k) + z^{\mathrm{T}}(k)z(k) - \eta w^{\mathrm{T}}(k)w(k) < 0 \tag{A.4}$$

holds for all $x(k)$ and $w(k)$ if

$$\begin{aligned} &\begin{bmatrix} A(\Theta) & B_w(\Theta) \end{bmatrix}^{\mathrm{T}} P(\Theta) \begin{bmatrix} A(\Theta) & B_w(\Theta) \end{bmatrix} \\ &+ \begin{bmatrix} C(\Theta) & D_w(\Theta) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} C(\Theta) & D_w(\Theta) \end{bmatrix} - \begin{bmatrix} P(\Theta) & 0 \\ 0 & \eta I_m \end{bmatrix} < 0. \end{aligned} \tag{A.5}$$

Let us transform the inequality (A.5) to the form

$$\begin{bmatrix} -P(\Theta) & 0 \\ 0 & -\eta I_m \end{bmatrix} - \begin{bmatrix} A(\Theta) & B_w(\Theta) \\ C(\Theta) & D_w(\Theta) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -P(\Theta) & 0 \\ 0 & -I_p \end{bmatrix} \begin{bmatrix} A(\Theta) & B_w(\Theta) \\ C(\Theta) & D_w(\Theta) \end{bmatrix} < 0, \tag{A.6}$$

where matrix $\begin{bmatrix} -P^{-1}(\Theta) & 0 \\ 0 & -I_p \end{bmatrix}$ is negative definite. Applying to the inequality (A.6) Schur complement, we have

$$\begin{bmatrix} -P(\Theta) & \star & \star & \star \\ 0 & -\eta I_m & \star & \star \\ A(\Theta) & B_w(\Theta) & -P^{-1}(\Theta) & \star \\ C(\Theta) & D_w(\Theta) & 0 & -I_p \end{bmatrix} < 0. \tag{A.7}$$

Fulfilling of the inequality (A.6) for zero input makes true inequalities of the form (A.4) for any $k \in \mathbb{Z}_+ \bigcup \{0\}$ and allows to sum up them from $k = 0$ to $k = \infty$. This implies the inequality

$$V(\infty) - V(0) + \sum_{k=0}^{\infty} z^{\mathrm{T}}(k)z(k) - \eta \sum_{k=0}^{\infty} w^{\mathrm{T}}(k)w(k) < 0. \tag{A.8}$$

For zero initial condition ($x(0) = 0$) $V(0) = 0$, assuming that $V(\infty) = 0$, inequality (A.8) transforms to the form

$$\sum_{k=0}^{\infty} z^{\mathrm{T}}(k)z(k) < \eta \sum_{k=0}^{\infty} w^{\mathrm{T}}(k)w(k).$$

Therefore,

$$\sup_{\Theta\in\mathfrak{Q}} \sup_{W\in\mathcal{L}_2} \frac{\sum_{k=0}^{\infty} z^{\mathrm{T}}(k)z(k)}{\sum_{k=0}^{\infty} w^{\mathrm{T}}(k)w(k)} < \eta. \tag{A.9}$$

Fulfilment of inequality (A.7) guarantees stability of the open loop system (1)–(2) and boundedness its $\mathcal{H}_\infty$-norm by scalar $\sqrt{\eta}$.

At the second step, it is necessary to find out conditions that guarantee the boundedness of the anisotropic norm for the mean anisotropy level $\overline{\mathbf{A}}(W) \leqslant a$ of input disturbances. Then conditions of anisotropic norm boundedness can be defined by anisotropy-based bounded real lemma [12] as follows:

$$-(\det(\Sigma(\Theta)))^{1/m} < -(1 - q\gamma^2)\mathrm{e}^{2a/m}, \tag{A.10}$$

$$\begin{bmatrix} A(\Theta)R(\Theta)A(\Theta) - R(\Theta) & A^{\mathrm{T}}(\Theta)R(\Theta)B_w(\Theta) \\ B_w^{\mathrm{T}}(\Theta)R(\Theta)A(\Theta) & B_w^{\mathrm{T}}(\Theta)R(\Theta)B_w(\Theta) - I_m \end{bmatrix} + q \begin{bmatrix} C^{\mathrm{T}}(\Theta) \\ D_w^{\mathrm{T}}(\Theta) \end{bmatrix} \begin{bmatrix} C(\Theta) & D_w(\Theta) \end{bmatrix} < 0, \tag{A.11}$$

where $q \in (0, \min(\gamma^{-2}, \|F_\Theta\|_\infty^{-2}))$, and $\Sigma(\Theta)$ defined by

$$\Sigma(\Theta) = (I_m - B_w^{\mathrm{T}}(\Theta)R(\Theta)B_w(\Theta) - qD_w^{\mathrm{T}}(\Theta)D_w(\Theta)). \tag{A.12}$$

Inequality (A.11) coincides with inequality (A.5) taking into account the change of variables $P(\Theta) = \eta R(\Theta)$ and $\eta = q^{-1}$. Thus, anisotropic norm of the system is bounded if inequalities (A.7) and (A.11) hold true.

Consider inequality (A.10) in detail. Taking into account introduced notations, it can be rewritten as

$$\eta - (\mathrm{e}^{-2a} \det(\eta I_m - B_w^{\mathrm{T}}(\Theta)P(\Theta)B_w(\Theta) - D_w^{\mathrm{T}}(\Theta)D_w(\Theta)))^{1/m} < \gamma^2. \tag{A.13}$$

Introducing new variable

$$\Psi(\Theta) < \eta I_m - B_w^{\mathrm{T}}(\Theta)P(\Theta)B_w(\Theta) - D_w^{\mathrm{T}}(\Theta)D_w(\Theta),$$

where $\Psi(\Theta) = \Psi(\Theta)^{\mathrm{T}} > 0$ [13], we ascertain inequality (A.13) fulfilled, if two following inequalities hold:

$$\eta - (\mathrm{e}^{-2a}\det(\Psi(\Theta)))^{1/m} < \gamma^2, \tag{A.14}$$

$$\Psi(\Theta) < \eta I_m - B_w^{\mathrm{T}}(\Theta)P(\Theta)B_w(\Theta) - D_w^{\mathrm{T}}(\Theta)D_w(\Theta). \tag{A.15}$$

Rewrite (A.15) as

$$\Psi(\Theta) - \eta I_m - \begin{bmatrix} B_w^{\mathrm{T}}(\Theta) & D_w^{\mathrm{T}}(\Theta) \end{bmatrix} \begin{bmatrix} -P(\Theta) & 0 \\ 0 & -I_p \end{bmatrix} \begin{bmatrix} B_w(\Theta) \\ D_w(\Theta) \end{bmatrix} < 0. \tag{A.16}$$

Applying Schur complement to the expression (A.16), we obtain

$$\begin{bmatrix} \Psi(\Theta) - \eta I_m & B_w^{\mathrm{T}}(\Theta) & D_w^{\mathrm{T}}(\Theta) \\ B_w(\Theta) & -P^{-1}(\Theta) & 0 \\ D_w(\Theta) & 0 & -I_p \end{bmatrix} < 0. \tag{A.17}$$

By right and left multiplying inequality (A.17) by matrix $\begin{bmatrix} I & 0 & 0 \\ 0 & G_1(\Theta) & 0 \\ 0 & 0 & I \end{bmatrix}$ and its transposed, we get

$$\begin{bmatrix} \Psi(\Theta) - \eta I_m & \star & \star \\ G_1(\Theta)B_w(\Theta) & \Lambda_1(\Theta) & \star \\ D_w(\Theta) & 0 & -I_p \end{bmatrix} < 0, \tag{A.18}$$

where $\Lambda_1(\Theta) = -G_1(\Theta)P^{-1}(\Theta)G_1^{\mathrm{T}}(\Theta)$.

Note that for $P(\Theta) > 0$ it follows from inequality

$$-(G_1(\Theta) - P(\Theta))^{\mathrm{T}}P^{-1}(\Theta)(G_1(\Theta) - P(\Theta)) \leqslant 0$$

that

$$-G_1(\Theta)P^{-1}(\Theta)G_1^{\mathrm{T}}(\Theta) \leqslant -G_1(\Theta) - G_1^{\mathrm{T}}(\Theta) + P(\Theta).$$

Introducing notation $L_1(\Theta) = -G_1(\Theta) - G_1^{\mathrm{T}}(\Theta) + P(\Theta)$ and replacing $\Lambda_1(\Theta)$ by $L_1(\Theta)$ at the inequality (A.18), we get inequality (14).

Let's get rid of the inversion of the matrix $P(\Theta)$ in the inequality (A.7).To do this, we introduce a new nonsingular matrix $G_2(\Theta)$. By right and left multiplying inequality (A.7) by nonsingular matrix

$$\begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & G_2(\Theta) & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \tag{A.19}$$

and its transposed respectively, we get:

$$\begin{bmatrix} -P(\Theta) & \star & \star & \star \\ 0 & -\eta I_m & \star & \star \\ G_2(\Theta)A(\Theta) & G_2(\Theta)B_w(\Theta) & \Lambda_2(\Theta) & \star \\ C(\Theta) & D_w(\Theta) & 0 & -I_p \end{bmatrix} < 0, \tag{A.20}$$

where

$$\Lambda_2(\Theta) = -G_2(\Theta)P^{-1}(\Theta)G_2^{\mathrm{T}}(\Theta). \tag{A.21}$$

Similar to previous case, we replace $\Lambda_2(\Theta)$ by expression $L_2(\Theta) = -G_2(\Theta) - G_2^{\mathrm{T}}(\Theta) + P(\Theta)$. As a result, we have expression (15).

Theorem 1 is proved.

Proof of Theorem 3. Define matrices $\Psi(\Theta)$, $G_1(\Theta)$, $G_2(\Theta)$, and $P(\Theta)$ in the form $\Psi(\Theta) = \Psi$, $G_1(\Theta) = \sum_{i=1}^{r} \theta_i G_{1i}$, $G_2(\Theta) = \sum_{i=1}^{r} \theta_i G_{2i}$, $P(\Theta) = \sum_{i=1}^{r} \theta_i P_i$. Rewrite inequalities (14) and (15) taking into account introduced assumptions.

It should be noted that the inequalities (14) and (15) contain blocks of constant matrices, parametric matrices and products of two parametric matrices. Taking into account the introduced appearance of parametric variables, and also taking into account the fact that $(\sum_{i=1}^{s} \theta_i)^2 = 1$, constant matrices can be written in the form

$$I_p = \left(\sum_{i=1}^{s} \theta_i\right)^2 I_p.$$

Because of identity $\sum_{j=1}^{r} \theta_j = 1$, parametric matrices can be rewritten as

$$\sum_{i=1}^{r} \theta_i A_i = \sum_{i=1}^{r} \theta_i \left(\sum_{j=1}^{r} \theta_j\right) A_i.$$

Expressions of the form $G_1(\Theta)B_w(\Theta)$ are written as follows:

$$G_1(\Theta)B_w(\Theta) = \sum_{i=1}^{r} \theta_i^2 (G_{1i}B_i) + \sum_{i=1}^{r} \sum_{i<j}^{r} \theta_i\theta_j (G_{1i}B_{wj} + G_{1j}B_{wi}).$$

Applying all above mentioned transformation to each element of inequalities (14) and (15), we get:

$$\begin{aligned} &\sum_{i=1}^{r} \theta_i^2 \begin{bmatrix} \Psi - \eta I_m & \star & \star \\ B_{wi} & -G_{1i} - G_{1i}^{\mathrm{T}} + P_i & \star \\ D_{wi} & 0 & -I_p \end{bmatrix} \\ &+ \sum_{i=1}^{r} \sum_{i<j}^{r} \theta_i\theta_j \left(\begin{bmatrix} \Psi - \eta I_m & \star & \star \\ G_{1i}B_{wj} & -G_{1i} - G_{1i}^{\mathrm{T}} + P_i & \star \\ D_{wj} & 0 & -I_p \end{bmatrix} \right. \\ &\left. + \begin{bmatrix} \Psi - \eta I_m & \star & \star \\ G_{1j}B_{wi} & -G_{1j} - G_{1j}^{\mathrm{T}} + P_j & \star \\ D_{wi} & 0 & -I_p \end{bmatrix} \right) < 0, \end{aligned}$$

$$\sum_{i=1}^{r}\theta_i^2\begin{bmatrix} -P_i & \star & \star & \star \\ 0 & -\eta I_m & \star & \star \\ G_{2i}A_i & G_{2i}B_{wi} & -G_{2i}-G_{2i}^{\mathrm{T}}+P_i & \star \\ C_i & D_{wi} & 0 & -I_p \end{bmatrix}$$

$$+\sum_{i=1}^{r}\sum_{i<j}^{r}\theta_i\theta_j\left(\begin{bmatrix} -P_i & \star & \star & \star \\ 0 & -\eta I_m & \star & \star \\ G_{2i}A_j & G_{2i}B_{wj} & -G_{2i}-G_{2i}^{\mathrm{T}}+P_i & \star \\ C_j & D_{wj} & 0 & -I_p \end{bmatrix}\right.$$

$$\left.+\begin{bmatrix} -P_j & \star & \star & \star \\ 0 & -\eta I_m & \star & \star \\ G_{2j}A_i & G_{2j}B_{wi} & -G_{2j}-G_{2j}^{\mathrm{T}}+P_j & \star \\ C_i & D_{wi} & 0 & -I_p \end{bmatrix}\right) < 0.$$

Since $\theta_i \geqslant 0$, $i=\overline{1,r}$, it obvious that inequalities (13)–(15) hold, when inequalities (19)–(23) hold.

Proof of Theorem 4. Let us consider inequalities (A.7) and (A.17), obtained in the proof of Theorem 1. Introduce new variable $\Phi(\Theta)=P^{-1}(\Theta)$, and fix parameter η and matrix Ψ. Then inequalities (A.7) and (A.17) will be rewritten as follows:

$$\begin{bmatrix} \Psi-\eta I_m & B_w^{\mathrm{T}}(\Theta) & D_w^{\mathrm{T}}(\Theta) \\ B_w(\Theta) & -\Phi(\Theta) & 0 \\ D_w(\Theta) & 0 & -I_p \end{bmatrix} < 0 \qquad \text{(A.22)}$$

and

$$\begin{bmatrix} -\Phi^{-1}(\Theta) & \star & \star & \star \\ 0 & -\gamma^2 I_m & \star & \star \\ A(\Theta) & B_w(\Theta) & -\Phi(\Theta) & \star \\ C(\Theta) & D_w(\Theta) & 0 & -I_p \end{bmatrix} < 0. \qquad \text{(A.23)}$$

The latest inequality contains matrix $\Phi^{-1}(\Theta)$. To get rid of it, we will left and right multiply inequality (A.23) by matrix

$$\begin{bmatrix} G^{\mathrm{T}}(\Theta) & 0 & 0 & 0 \\ 0 & I_m & 0 & 0 \\ 0 & 0 & I_n & 0 \\ 0 & 0 & 0 & I_p \end{bmatrix}$$

and its transposed respectively. It results to:

$$\begin{bmatrix} \Lambda(\Theta) & \star & \star & \star \\ 0 & -\eta I_m & \star & \star \\ A(\Theta)G(\Theta) & B_w(\Theta) & -\Phi(\Theta) & \star \\ C(\Theta)G(\Theta) & D_w(\Theta) & 0 & -I_p \end{bmatrix} < 0, \qquad \text{(A.24)}$$

where $\Lambda(\Theta)=-G^{\mathrm{T}}(\Theta)\Phi^{-1}(\Theta)G(\Theta)$.

Note that $\Phi(\Theta) > 0$, therefore fulfilment of inequality

$$-(G(\Theta) - \Phi(\Theta))^{\mathrm{T}}\Phi^{-1}(\Theta)(G(\Theta) - \Phi(\Theta)) \leqslant 0$$

results to $-G^{\mathrm{T}}(\Theta)\Phi^{-1}(\Theta)G(\Theta) \leqslant -G(\Theta) - G^{\mathrm{T}}(\Theta) + \Phi(\Theta)$. From the latter it follows that inequality (A.24) holds, if inequality (A.23) holds.

Consider matrix $\Phi(\Theta)$ be appeared in the form $\Phi(\Theta) = \sum_{i=1}^{r}\theta_i\Phi_i$ taking into account expressions for parametric uncertainties (3)–(4). Then inequalities (A.22) and (A.24) take form:

$$\sum_{i=1}^{r}\theta_i\begin{bmatrix}\Psi-\eta I_m & \star & \star\\ B_{wi} & -\Phi_i & \star\\ D_{wi} & 0 & -I_p\end{bmatrix} < 0, \tag{A.25}$$

$$\sum_{i=1}^{r}\theta_i^2\begin{bmatrix}-G_i-G_i^{\mathrm{T}}+\Phi_i & \star & \star & \star\\ 0 & -\eta I_m & \star & \star\\ A_iG_i & B_{wi} & -\Phi_i & \star\\ C_iG_i & D_{wi} & 0 & -I_p\end{bmatrix}$$
$$+\sum_{i=1}^{r}\sum_{i<j}^{r}\theta_i\theta_j\left(\begin{bmatrix}-G_i-G_i^{\mathrm{T}}+\Phi_i & \star & \star & \star\\ 0 & -\eta I_m & \star & \star\\ A_jG_i & B_{wj} & -\Phi_i & \star\\ C_jG_i & D_{wj} & 0 & -I_p\end{bmatrix}\right. \tag{A.26}$$
$$\left.+\begin{bmatrix}-G_j-G_j^{\mathrm{T}}+\Phi_j & \star & \star & \star\\ 0 & -\eta I_m & \star & \star\\ A_iG_j & B_{wi} & -\Phi_j & \star\\ C_iG_j & D_{wi} & 0 & -I_p\end{bmatrix}\right) < 0.$$

Note that inequality (A.26) can be obtained using property (4) and considering that $(\sum_{i=1}^{r}\theta_i)^2 = 1$. Obviously, fulfilment inequalities (25)–(27) automatically leads to fulfilment inequalities (A.25) and (A.26), that completes the proof.

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